

Name: \_\_\_\_\_

Section: \_\_\_\_\_

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**IMPORTANT:** All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. (15 pts) For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. For each T/F question, write a careful and clear justification or describe a counterexample.

(a) If  $f$  and  $g$  are increasing on an interval  $(a,b)$  then  $fg$  is also increasing on  $(a,b)$ . (a) T F  
Give justification:

(b)  $\frac{d}{dx}(\ln(10)) = \frac{1}{10}$  (b) T F  
Give justification:

(c) If  $f$  has an absolute minimum value at  $x = c$  then  $f'(c) = 0$ . (c) T F  
Give Justification:

(d) If  $f, f', f''$  are continuous then the inflection points of  $f$  are the local extrema of  $f'$ . (d) T F  
Give Justification:

(e) A function defined on all points of a closed interval  $[a, b]$  always has a global maximum and a global minimum on  $[a, b]$ . (e) T F  
Give Justification:

2. (20 pts) For the function  $f(x) = \left(\frac{1}{4}\right)x^4 - 2x^3 + \left(\frac{9}{2}\right)x^2 + 1$ , use the techniques of calculus to answer the following. Show your work and proper justification for your answers.
- (a) Find  $f'(x)$ .
- (b) Determine all critical points of  $f$ . Classify each as a local maximum, a local minimum, or neither.
- (c) Find the intervals on which the function is increasing.
- (d) Determine any global maxima or minima (if any).
- (e) Find any inflection points and the intervals on which the graph of  $f$  is concave down.

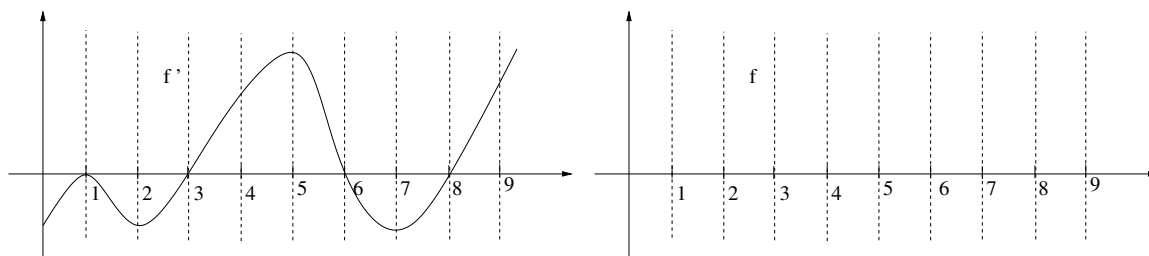
3. (10 pts) (a) Find the tangent line approximation to  $f(x) = x^{2/3}$  at  $a = 8$ .

(b) Use your answers to (a) to obtain a linear approximation estimating the value of  $(7.98)^{2/3}$ .

(c) Will this be an overestimate or an underestimate? Explain your reasoning. [Hint: Look at the shape of the graph.]

4. (10 pts) Use L'Hopital's rule to evaluate  $\lim_{x \rightarrow 0} \frac{e^{3x} - 3x - 1}{x^2}$

5. (15 pts) The graph of  $f'$  is shown below. (The graph of  $f$  is not shown.) Use the graph of  $f'$  to answer the following questions.
- On which intervals, if any, is  $f$  increasing?
  - At which values of  $x$ , if any, does  $f$  have a local maximum? A local minimum?
  - On which intervals, if any, is  $f$  concave up?
  - Which values of  $x$ , if any, correspond to inflection points on the graph of  $f$ ?
  - Assume that  $f(0) = 0$ . Sketch a graph of  $f$ . (Your graph need only have the right general shape. You do not need to put units on the vertical axis.)



6. (10 pts) A man 6 feet tall is walking away from a street light 12 feet high. If the man is walking at a rate of 1.8 ft/min, how fast is the length of his shadow increasing? At what rate is the top end of the shadow moving?

7. (10 pts) A bacteria culture contains 300 cells initially and grows at a rate proportional to its size. After two hours the population has increased to 420 cells.
- (a) Find the number of bacteria after  $t$  hours.
- (b) Find the rate of growth after 4 hours.
- (c) When will the population reach 15,000?
8. (10 pts) A rectangular building is to cover 20,000 square feet. Building lots are rectangular. Zoning regulations require 20 foot borders in front and back of the building and a 10 foot border on each side. Use the optimization techniques of calculus to find the dimensions of the smallest piece of property on which the building can be legally built.