University of Connecticut
Department of Mathematics

Math 1131 — Sample Final Exam — Fall 2015

Name: ____________________

This sample exam is just a *guide* to prepare for the actual exam. Questions on the actual exam may or may not be of the same type, nature, or even points. Don't prepare only by taking this sample exam. You also should review your class notes, homework and quizzes on WebAssign, quizzes in discussion section, and worksheets.

The actual exam could have a question from any topic covered in the course.

Read This First!

- Please read each question carefully. Other than the question of true/false items, show **all** work clearly in the space provided. In order to receive full credit on a problem, solution methods must be complete, logical and understandable.
- Answers must be clearly labeled in the spaces provided after each question. Please cross out or fully erase any work that you do not want graded. The point value of each question is indicated after its statement. No books or other references are permitted.
- Give any numerical answers in exact form, **not as approximations**. For example, one-third is $\frac{1}{3}$, not .33 or .33333. And one-half of π is $\frac{1}{2}\pi$, not 1.57 or 1.57079.
- Turn smart phones, cell phones, and other electronic devices **off** (not just in sleep mode) and store them away.
- Calculators are allowed but you must show all your work in order to receive credit on the problem.
- If you finish early then you can hand in your exam early.

Grading - For Administrative Use Only

Question:	1	2	3	4	5	6	7	8	9	10	11	Total
Points:	15	8	6	8	10	9	10	8	9	9	8	100
Score:												

1. If the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, write a careful and clear justification or a counterexample.

   (a) For every number x, $\ln(2^{3x}) = 3\ln(2^x)$. T F [3]

   Justification:

   (b) $\frac{d}{dx}(\sin^3 x) = 3\sin^2 x \cos x$ T F [3]

   Justification:

   (c) If $f'(x) = \ln x$ for all $x > 0$ then $(f(x^2))' = 4x \ln x$ for all $x > 0$. T F [3]

   Justification:

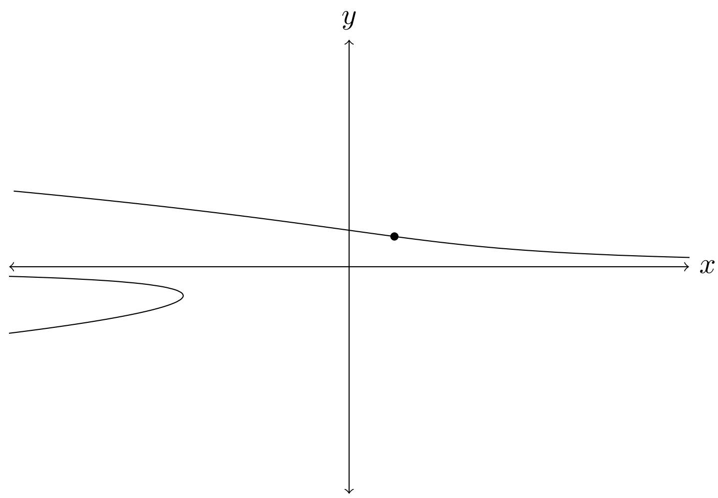
   (d) $\int \frac{2x}{x^2+1}\, dx = \ln(x^2+1) + C$. T F [3]

   Justification:

   (e) If $\frac{d}{dx}\int_2^5 t^2\, dt = 5^2$. T F [3]

   Justification:

2. Use calculus to find the equation of the tangent line to the curve $xy + y^3 = 14$ at the point $(3, 2)$. [8]



3. Use calculus to compute the following limits. If the limit does not exist, write your final answer as DNE.

   (a) $\lim_{x\to 2} \frac{4^x - x^4}{x^2 - 4}$ [3]

   (b) $\lim_{x\to\infty} \left(1 + \frac{100}{x}\right)^x$ [3]

4. A helicopter rises vertically so that at time t its height is $h(t) = t^2 + t$, where t is measured in seconds and $h(t)$ is measured in meters. At the time when the height of the helicopter is 20 m, what is its velocity (in m/sec) and acceleration (in m/sec^2)? (Hint: First find the time when the height is 20 m.) [8]

5. Let $f(x) = x^4 - 8x^2$. Use calculus to find the open intervals on which f is increasing or decreasing, the local maximum and minimum values of f, the intervals of concavity and the inflection points. [10]

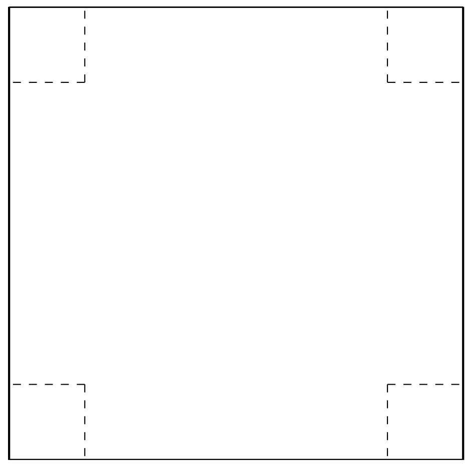
6. (a) Newton's Law of Cooling says that when an object with temperature $T(t)$ at time t is in a room with constant surrounding temperature T_s, the rate of change of $T(t)$ is described by the differential equation [3]

$$\frac{dT}{dt} = k(T - T_s)$$

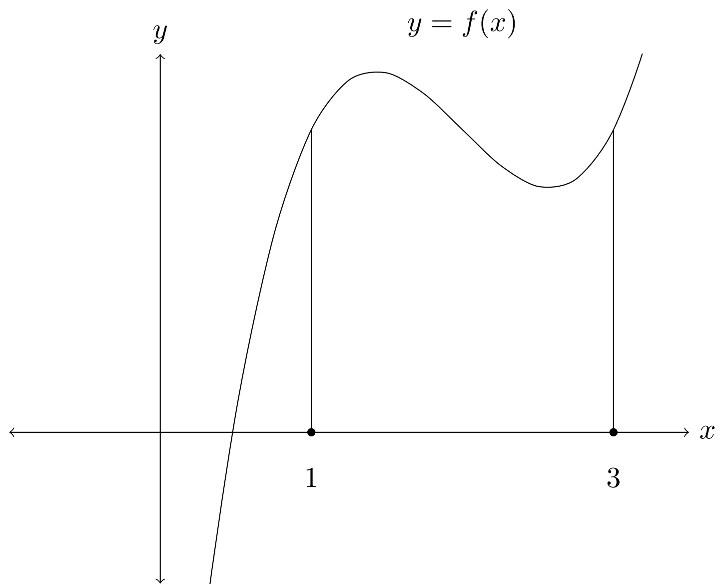
for some constant k. For what change of variables can this be converted into the simpler differential equation $\frac{dy}{dt} = ky$?

(b) A dead body is found in a room where the thermostat is 65^o. (All degrees are Fahrenheit.) The body's temperature is measured to be 85^o, and an hour later it is 83^o. Use the temperature formula from Newton's Law of Cooling to estimate how long ago the body died, measured in minutes from when it was first found. Assume the temperature of the person, when last alive, was 98.6^o. [6]

7. After cutting out square corners from a square piece of cardboard, as in the figure below, what remains can be folded up to form a rectangular box with an open top. Find the dimensions of the original rectangular cardboard and the side length of the cut corners that produces a box with volume 36 in^3 and requiring the least amount of initial cardboard. [10]



8. In the figure below, draw rectangles in the Riemann sum approximation to $\int_1^3 f(x)\,dx$ using 4 rectangles with **right** endpoints. Label numerically where the sides of the rectangles meet the x-axis. [8]



9. Compute the following indefinite integrals using rules of integration.

   (a) $\displaystyle\int \left(e^{kx} + \frac{1}{x^2}\right) dx$, where k is a nonzero constant. [3]

   (b) $\displaystyle\int x\sqrt{x^2 - 5}\, dx$ [3]

   (c) $\displaystyle\int (\sin x)(\cos^2 x)\, dx$ [3]

10. (Definite integrals)

(a) Compute $\int_0^2 e^{3x}\,dx$. [3]

(b) Compute $\int_1^b (x^3 - x)\,dx$, where b is a constant. [3]

(c) Express $\int_1^3 x\sqrt{x^2-1}\,dx$ as a definite integral in terms of $u = x^2 - 1$, but do not evaluate the integral. [3]

11. Set up, but do **NOT** evaluate, a definite integral for the following geometric quantities. In each case draw graphs to help you determine the correct integral.

    (a) The area between $y = x^2 - 1$ and $y = 1 - x^2$ for $-1 \leq x \leq 1$. [4]

    (b) The solid whose base is bounded by $y = x^2 - 1$ and $y = 1 - x^2$ with $-1 \leq x \leq 1$ and whose cross-sections parallel to the y-axis are equilateral triangles. [4]