\[
C(15, 2) = \frac{15 \cdot 14}{2} = 105
\]

\[
C(6, 2) = 15
\]

\[
P(2 \text{ def. tens}) = \frac{C(6, 2)}{C(15, 2)} = \frac{15}{105} = \frac{1}{7}
\]

\[
C(n, r) = \frac{n!}{(n-r)!}
\]

\[
\begin{align*}
C(15, 2) &= \frac{15!}{(13)!} = \frac{15 \cdot 14}{2 \cdot 13!} = \frac{15 \cdot 14 \cdot 13!}{2 \cdot 13!} = \frac{15 \cdot 14}{2} = 105 \\
C(9, 3) &= 84 \\
C(6, 2) &= 15
\end{align*}
\]

\[
\frac{C(6, 2) \cdot C(9, 3)}{C(15, 5)} = \frac{84 \times 15}{3003} = 0.42
\]
5. balls

OR

3W and 2R
4W and 1R

\[ \binom{4}{1,3} \cdot \binom{6}{2} = 60 \] \[ \binom{4}{4} \cdot \binom{6}{1} + \frac{6}{66} = \]

\[ P(E) = \frac{66}{252} \]

10. Standard deck of 52 cards

We are drawing 2 cards

Sample = \( \binom{52}{2} = 1326 \)

Same suit = \( \binom{13}{2} = 78 \)

\[ P(E) = \frac{78}{1326} = \frac{1}{17} \]

\( \binom{52}{2} = 1326 \)

\( \binom{40}{2} = 780 \)

\( \frac{780}{1326} = \frac{10}{17} \)
Toss a coin six times

3 Heads

\[ C(6, 3) = 20 \]

\[ P(\text{Head exactly 3 times}) = \frac{20}{2^6} = \frac{20}{64} \]

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Page 276

2. Urn has 10 balls.

4 white, 6 red

\[ C(10, 5) = 252 \]

3 W \[ C(4, 3) = 4 \]

2 R \[ C(6, 2) = 15 \]

\[ P(3W \& 2R) = \frac{C(4, 3) \cdot C(6, 2)}{C(10, 5)} = \frac{4 \times 15}{252} = \frac{5}{21} \]
20 \rightarrow 3 \quad C(20, 3) \rightarrow \text{sample}

10 \rightarrow \text{Above} \quad C(10, 3) \rightarrow \quad \frac{C(10, 3)}{C(20, 3)} = \frac{2}{19}

\begin{tikzpicture}
  \node (F) {F} child {node (N) {N} child {node {0.09} child {node {0.09} child {node {0.829}}}} child {node {0.91} child {node {0.93} child {node {0.171}}}}};
\end{tikzpicture}

\[ P(F|+) = \frac{P(F \cap +)}{P(F) \cdot P(+|F) + P(N) \cdot P(+|N)} = \frac{(0.171)(0.93)}{(0.171)(0.93) + (0.829)(0.09)} \]
5.4 Bernoulli Trials

- Flip a coin and see if it is head or tails
- Test a transistor to see if it is defective or not
- Examine a patient to see if a particular disease is present or not
- Take a free throw in basketball and make the basket or not

Fundamental Assumption for Bernoulli Trials

Successive Bernoulli trials are independent from each other

Example 1

S - making the basket
F - not making the basket

A typical sequence could be

SSFSFSSFFS

\[
\left( \frac{2}{3} \right) \left( \frac{2}{3} \right) \left( \frac{1}{3} \right) \left( \frac{4}{3} \right) \left( \frac{2}{3} \right) \left( \frac{4}{3} \right) \left( \frac{2}{3} \right) \left( \frac{4}{3} \right) \left( \frac{2}{3} \right) \left( \frac{4}{3} \right) = \left( \frac{2}{3} \right)^4 \left( \frac{1}{3} \right)^4
\]

Also count the number of ways these 6 success occur

\[ C \left( 10, 6 \right) \]
\[ C(10, 6) \left( \frac{2}{3} \right)^6 \left( \frac{1}{3} \right)^4 = 0.228 \]

Formula:

\[ C(n, k) p^k q^{n-k} \]

- \( k \): no of successes in \( n \) trials
- \( n \): no of trials
- \( p \): probability of success
- \( q \): probability of failure = 1 - \( p \)

Baseball hits
\[ n = 6, \ k = 4 \quad p = 0.5 \quad q = 1 - p = 0.5 \]
\[ C(6, 4) \cdot (0.5)^4 \cdot (0.5)^2 = 0.2344. \]

\[ n = 10 \]

head = success = \( p = 0.5 \)

Tail = failure = \( q = 0.5 \).

\( k = 7, 8, 9, 10 \)

\[ C(10, k) = (p)^k \cdot (q)^{n-k} \]

\[ = C(10, 7) \cdot (0.5)^7 \cdot (0.5)^3 \]
\[ + C(10, 8) \cdot (0.5)^8 \cdot (0.5)^2 \]
\[ + C(10, 9) \cdot (0.5)^9 \cdot (0.5)^1 \]
\[ + C(10, 10) \cdot (0.5)^{10} \cdot (0.5)^0 \]

\[ = 0.172. \]

\( P(E) = 0.6 \quad q = 0.4. \)

\[ n = 7 \quad k = 3 \]

\[ C(7, 3) \cdot (0.6)^3 \cdot (0.4)^4 = 0.1935. \]
\[ S \text{ - Success of making the free throw in basket} \]
\[ F \text{ - Failure} \]
\[ \begin{align*}
(6/10) & \quad (4/10) \\
\end{align*} \]
\[ p = \frac{6}{10} \quad q = \frac{4}{10} = 1 - p \]

\[ SSFSFFSSFFSS \]
\[ \left( \frac{6}{10} \right) \left( \frac{6}{10} \right) \left( \frac{4}{10} \right) \left( \frac{6}{10} \right) \left( \frac{6}{10} \right) \left( \frac{4}{10} \right) \left( \frac{4}{10} \right) \left( \frac{6}{10} \right) \]
\[ = \left( \frac{6}{10} \right)^6 \cdot \left( \frac{4}{10} \right)^4 \]

\[ P(\text{Six go in exactly}) = C(10,6) \cdot \left( \frac{6}{10} \right)^6 \cdot \left( \frac{4}{10} \right)^4 \]

\[ = C(n,k) \cdot (p)^k \cdot (q)^{n-k} \]
\[ n = 6 \quad k = 4 \quad p = 0.5 \quad q = 0.5 \]

\[ P(\text{exactly 4}) = \binom{n}{k} \cdot (p)^k \cdot (q)^{n-k} \]
\[ = \binom{6}{4} \cdot (0.5)^4 \cdot (0.5)^2 \]
\[ = 0.2344 \]

16. Head - Success = 0.5 = p
Tail - Failure = 0.5 = q

\[ n = 10 \quad p = 0.5 \quad q = 0.5 \]

\[ \binom{10}{7} \cdot (0.5)^7 \cdot (0.5)^3 + \binom{10}{8} \cdot (0.5)^8 \cdot (0.5)^2 + \binom{10}{9} \cdot (0.5)^9 \cdot (0.5)^1 + \binom{10}{10} \cdot (0.5)^0 \cdot (0.5)^0 = 0.172 \]

20. \[ P(E) = 0.6 \quad q = 0.4 \]

\[ n = 7 \quad k = 3 \]

\[ \binom{7}{3} \cdot (0.6)^3 \cdot (0.4)^4 = 0.1935 \]
\[ P(B \mid \text{DND}) = \frac{P(B \cap \text{DND})}{P(B) \cdot P(\text{DND} \mid B) + P(\text{ND}) \cdot P(\text{DND} \mid \text{ND})} \]

\[ = \frac{0.1 \cdot [0.1(0.9)]}{(0.1)(0.1)(0.9) + (0.9)(0.01)(0.99)} = 0.5025. \]

Also can use combination.

\[ = 0.1 \cdot \frac{C(101, 1) \cdot C(90, 1)}{C(100, 2)} \]

\[ = 0.1 \cdot \frac{(101) \cdot C(90, 1)}{C(100, 2)} + 0.9 \cdot \frac{(10)(1) \cdot C(99, 1)}{C(100, 2)} \]

\[ = 0.5025. \]
10 boxes  
1 box $\rightarrow$ 10 -D- defective

\[ D = \frac{10}{100} = 0.1 \]

\[ ND = 0.9 \]

9 boxes $\rightarrow$ 1 -D

\[ D = \frac{1}{100} \quad ND = \frac{99}{100} \]

\[
P(B \mid DND) = \frac{P(B \cap DND)}{P(B) \cdot P(DND \mid B) + P(R) \cdot P(DND \mid R)}
\]

\[
= \frac{(0.1)(0.1)(0.9)}{(0.1)(0.1)(0.9) + (0.9)(0.01)(0.99)} = 0.5025
\]
\( p = \frac{1}{20} \)
\( q = \frac{19}{20} \)
\( n = 12 \)
\( k = 4 \).

\[
\binom{12}{4} \cdot \left( \frac{1}{20} \right)^4 \left( \frac{19}{20} \right)^8
\]

\[
= 0.00205.
\]

\( q = \frac{1}{5} \)
\( p = \frac{4}{5} \)
\( n = 20 \)
\( k = 1 \) or 2.

\[
\binom{20}{1} \left( \frac{4}{5} \right)^1 \left( \frac{1}{5} \right)^{19}
\]

\[
+ \binom{20}{2} \left( \frac{4}{5} \right)^2 \left( \frac{1}{5} \right)^{18}
\]

\[
\approx 3.27 \times 10^{-11}
\]
F = \frac{1}{5} = \alpha

S = \frac{4}{5} = \beta

n = 20

At most 2

meaning \begin{array} {c}
1 \text{ OR } 2 \\
K = 1 \text{ OR } K = 2 \\
\end{array}

\begin{align*}
C(20, 1) \cdot \left(\frac{4}{5}\right)^1 \left(\frac{1}{5}\right)^{19} + C(20, 2) \left(\frac{4}{5}\right)^2 \left(\frac{1}{5}\right)^{18} \\
= 3.27 \times 10^{-11}
\end{align*}