4.5 Rules of Probability

\[ 0 \leq P(E) \leq 1 \]
\[ P(\emptyset) = 0 \]
\[ P(S) = 1 \]

Union Rule of Probability.

\[ P(E \cup F) = P(E) + P(F) - P(\text{EN}F) \]

\[ n(S) = 52 \quad P(R) = \frac{26}{52} \]
\[ n(R) = 26 \quad P(K) = \frac{4}{52} \]
\[ n(K) = 4 \]
\[ P(R \cap K) = \frac{2}{52} \]

\[ P(R \cup K) = \frac{26}{52} + \frac{4}{52} - \frac{2}{52} = \frac{28}{52} \]
$S = \{ (1, 1) \ (1, 2) \ (1, 3) \ (1, 4) \ (1, 5) \ (1, 6) \}$

$E = \{ (1, 1) \ (1, 2) \ (1, 3) \ (1, 4) \ (1, 5) \ (1, 6) \ \ (2, 1) \ (2, 2) \ (2, 3) \ (2, 4) \ (2, 5) \ (2, 6) \}$

$F = \{ (1, 5) \ (2, 4) \ (3, 3) \ (4, 2) \ (5, 1) \}$

$E \cap F = \{ (1, 5) \ (2, 4) \}$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

$= \frac{12}{36} + \frac{5}{36} - \frac{2}{36} = \frac{15}{36}$

$P(E \cup F) = P(E) + P(F)$

$P(E^c) = 1 - P(E)$
The odds in favor of an event \( E \) are defined to be the ratio of \( P(E) \) to \( P(E^c) \):

\[
\frac{P(E)}{P(E^c)} = \frac{P(E)}{1 - P(E)}
\]

\[
\frac{a}{b} = \frac{P(E)}{1 - P(E)}
\]

\[
a(1 - P(E)) = b P(E)
\]

\[
a - aP(E) = b P(E)
\]

\[
a = aP(E) + b P(E)
\]

\[
a = P(E) [a + b]
\]

\[
P(E) = \frac{a}{a + b}
\]
\[
\frac{a}{b} = 2 \quad \Rightarrow \quad p(E) = \frac{3}{2} \quad \Rightarrow \quad p(E) = \frac{a}{a+b} = \frac{3}{5} \quad \Rightarrow \quad a = 3 \quad \text{and} \quad b = 2
\]