

**Integration Gateway Test I**

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**Find the following integrals. Assume  $a$  is a constant.**

1.  $\int \left( x^2 + \frac{1}{x^2} - \frac{1}{\sqrt{x}} \right) dx$  \_\_\_\_\_

2.  $\int \left( \sin t - \frac{\cos t}{2} \right) dt$  \_\_\_\_\_

3.  $\int \left( e^x - \frac{1}{x} \right) dx$  \_\_\_\_\_

4.  $\int 3(x+1)^2 dx$  \_\_\_\_\_

5.  $\int \sqrt{x}(x+a) dx$  \_\_\_\_\_

6.  $\int \frac{y^2 + 1/y}{y} dy$  \_\_\_\_\_

7.  $\int \sin(\theta + 2) d\theta$  \_\_\_\_\_

8.  $\int ae^{(x+1)} dx$  \_\_\_\_\_

9.  $\int \frac{dx}{x^2 + 1}$  \_\_\_\_\_

10.  $\int \frac{dt}{\sqrt{1-t^2}}$  \_\_\_\_\_

### Solutions to Integration Gateway Test I

$$1. \int \left( x^2 + \frac{1}{x^2} - \frac{1}{\sqrt{x}} \right) dx = \frac{x^3}{3} - \frac{1}{x} - 2x^{-1/2} + C$$

$$2. \int \left( \sin t - \frac{\cos t}{2} \right) dt = -\cos t - \frac{\sin t}{2} + C$$

$$3. \int \left( e^x - \frac{1}{x} \right) dx = e^x - \ln|x| + C$$

$$4. \int 3(x+1)^2 dx = (x+1)^3 + C$$

5. Multiplying out, we obtain

$$\begin{aligned} \int \sqrt{x}(x+a) dx &= \int x^{3/2} + ax^{1/2} dx \\ &= \frac{2}{5}x^{5/2} + \frac{2}{3}ax^{3/2} + C \end{aligned}$$

6. Dividing through by  $y$  yields

$$\begin{aligned} \int \frac{y^2 + 1/y}{y} dy &= \int \left( y + \frac{1}{y^2} \right) dy \\ &= \frac{y^2}{2} - \frac{1}{y} + C \end{aligned}$$

7. Substituting  $w = \theta + 2$ ,  $dw = d\theta$ , we have

$$\int \sin(\theta + 2) d\theta = \int \sin w dw = -\cos w + C = -\cos(\theta + 2) + C$$

8. Substituting  $w = x + 1$ ,  $dw = dx$ , we have

$$\int ae^{(x+1)} dx = a \int e^w dw = ae^w + C = ae^{(x+1)} + C$$

$$9. \int \frac{dx}{x^2 + 1} = \arctan x + C$$

$$10. \int \frac{dt}{\sqrt{1-t^2}} = \arcsin t + C$$