

Name: \_\_\_\_\_

Section: \_\_\_\_\_

**IMPORTANT:** All answers must include either supporting work or an explanation of your reasoning. These elements are considered part of the answer and will be graded.

1. For each part, if the statement is always true, circle the printed capital T. If the statement is sometimes false, circle the printed capital F. In each case, provide a careful and clear justification or a counterexample.

(a) The graph of  $r = \cos \theta$  is a circle. (a) T F

Justification:

(b) If the  $x = f(t), y = g(t)$  is a parametrization of the unit circle centered at the origin, then the integral  $\int_0^{2\pi} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt$  is equal to  $2\pi$ . (b) T F

Justification:

(c) The location of the center of mass of a system of three masses on the  $x$ -axis does not change if all the three masses are doubled. (c) T F

Justification:

(d) The sequence  $s_n = \frac{\sin n}{n}$  diverges. (d) T F

Justification:

(e) The series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$  converges since  $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$ . (e) T F

Justification:

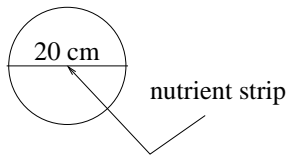
2. Consider the statement: "The series  $\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$  converges."

(a) Is this statement true or false.

(b) Justify your answer with a few sentences.

3. Write a definite integral that represents the area inside of the circle  $r = 1$  and outside of the cardioid  $r = 1 + \sin(\theta)$ .
4. A rectangular water tank has length 20 ft, width 8 ft, and depth 16 ft. If the tank is full, how much work does it take to pump all the water out? (Water weights 62.4 lb/ft<sup>3</sup>.)

5. A thin strip of nutrients 20 cm long is placed in a circular petri dish of radius 10 cm, as shown. The population density of bacteria in the disk after 3 hours is given by  $\frac{80}{D+4}$  bacteria/cm<sup>2</sup> where  $D$  is the distance (in cm) to the nutrient strip. Write a definite integral that gives number of bacteria in the dish 3 hours after the nutrient strip is introduced. You need not evaluate.



6. The density of cars (in cars per mile) down a 20-mile stretch of the Massachusetts Turnpike starting at a toll plaza is given by  $\rho(x) = 500 + 100 \sin(\pi x)$ , where  $x$  is the distance in miles from the toll plaza and  $0 \leq x \leq 20$ . Write a definite integral that expresses the total number of cars down the 20-mile stretch and evaluate it.
7. Find the sum of the infinite product  $5 + 2(.6) + 2(.6)^2 + 2(.6)^3 + 2(.6)^4 \dots + 2(.6)^n \dots$
8. Does the infinite series  $\sum_{n=1}^{\infty} \frac{2^n n^2}{e^n}$  converge? Explain why or why not.
9. Does the infinite series  $\sum_{n=1}^{\infty} \frac{n^3}{n^5 + 1}$  converge? Explain why or why not.
10. (a) Show that  $\int_1^{\infty} \frac{1}{x} dx$  does not converge.  
 (b) Use Part (a) to show that the harmonic series  $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \dots$  does not converge. (Hint: Consider a left hand sum of  $f(x) = \frac{1}{x}$  with  $\Delta x = 1$ .)
11. Use appropriate tests to decide whether or not the series  $\sum_{n=1}^{\infty} \frac{\sin n}{\sqrt{n^3 + 1}}$  converges absolutely, converges or diverges.