Calculus for Business & Economics

MIDTERM EXAMINATION 2

Name: ________________________________

Student ID number: ____________________    Section: _________

Instructor’s name: ________________________________

• Make sure that you have a complete exam: 8 pages including this one. The last page is for scrap.

• No books or other references are permitted. Calculators are permitted, but they must not be able to perform symbolic operations or store symbolic formulas.

• Show all your work clearly in the space provided. For full credit, solution methods must be complete, logical and understandable, and must involve only techniques and results developed thus far in this course (and its prerequisites). Answers must be clearly labeled, must give the information asked for, and must follow logically from earlier work. Be sure to read the question carefully! Work done outside the question’s space can be considered only if there are clear and explicit directions to it within the question’s workspace. Mark out (or fully erase) any work that you do not want graded.

Do not write anything on this part of the cover page.

1a,b,c,d _____    5. _________    10. _______
1e,f,g _____    6. _________    11. _______
2. _______    7. _________    12. _______
3. _________    8. _________
4. _________    9. _________

TOTAL SCORE: __________
1. Differentiate each of the following (5 pts. each, 35 points total):

   a) \( y = 3^x + \ln 2 \)  \( \frac{dy}{dx} = \) __________

   b) \( f(x) = (4x^3 + \sqrt{x} + 5)^2 \)  \( f'(x) = \) __________

   c) \( g(t) = (3t^2 + 1) \cdot \sqrt{t^2} \)  \( D_t(g(t)) = \) __________

   d) \( f(x) = \frac{1}{x^2} + \ln x + e^{x^2} \)  \( f'(x) = \) __________
1 continued

e) \( f(x) = 3^x + \log(x^2 + 1) \)  \hspace{1cm} f'(x) = ____________

f) \( y = \frac{3x + 2}{4x - 1} \)  \hspace{1cm} y' = ____________

g) \( y = \ln\left(\frac{x^2}{3x^2 + 1}\right) \)  \hspace{1cm} y' = ____________

2. (6 pts.) Find the following limits if they exist; if not, explain. Show all your work.

a) \( \lim_{x \to -\infty} \left(5 - 7e^{-x^2}\right) \)  ____________

b) \( \lim_{x \to \infty} \frac{1 + 3x - x^3}{8x^2 - 2x^3} \)  ____________
3. Consider the function $f(x) = 3x^5 - 20x^3 + 5$.

a) (10 pts.) Using calculus, find: (show all work needed to support your answers)
   • all critical numbers,
   • where the function is increasing and where it’s decreasing
   • where the graph is concave up and where it’s concave down
     You may leave your answers expressed with number lines or intervals.
   • all relative maximum and relative minimum values and where they are attained

b) (5 pts.) Sketch a graph of the function $f(x) = 3x^5 - 20x^3 + 5$ using all the information you arrived at above in part (a).
4. Consider the graph of the derivative $f'(x)$ of $f$ over $[0,4]$ given below:

![Graph of $f'(x)$]

a) (2 pts.) Find the $x$-values where the original function $f(x)$ has horizontal tangents. **Remember:** the function pictured above is $f'(x)$, **not** $f(x)$.

b) (2 pts.) Find the intervals over which the original function $f(x)$ is increasing and those intervals over which $f(x)$ is decreasing. **Remember:** the function pictured above is $f'(x)$, **not** $f(x)$.

c) (2 pts.) Using your answer to part (b), find the $x$-values at which $f(x)$ attains a relative maximum value and at which $f(x)$ attains a relative minimum value. **Remember:** the function pictured above is $f'(x)$, **not** $f(x)$.

d) (2 pts.) Find the intervals over which the original function $f(x)$ is concave up and over which $f(x)$ is concave down. **Remember:** the function pictured above is $f'(x)$, **not** $f(x)$.

e) (2 pts.) Using your answer to part (d), find the $x$-values where the original function $f(x)$ has an inflection point. **Remember:** the function pictured above is $f'(x)$, **not** $f(x)$. 
5. (8 pts.) The average cost per item function $A$ for producing $x$ hundreds of items in a month is given by $A(x) = \frac{4x^3 + 640}{x}$. How many items should be produced in a month in order to minimize the average cost per item? (Show all the calculus work needed to support and justify your conclusions.)

6. Hercules Films is deciding on the price of the video release of its film *Bride of the Son of Frankenstein*. Marketing estimates that the demand, at a price of $p$ dollars, is $x = 200,000 - 10,000p$ copies.

   a) (5 pts.) Find the elasticity of demand. Show your work.

   b) (2 pts.) Find whether the market is elastic, inelastic, or neither at the price level of $\$12$ per copy. Show your work.

   c) (2 pts.) At the price level of $\$12$ per copy, should the company raise the price a bit more to raise revenue or will doing so actually decrease revenue? Explain.
7. Suppose that the differentiable function \( y = f(x) \) satisfies the equation

\[ y^2 - xy + 2 = x^2y^2 \]

a) (6 pts.) Find \( \frac{dy}{dx} \).

b) (3 pts.) Find the equation of the tangent line to the graph of \( y = f(x) \) at the point \((1, 2)\).

8. a) (2 pts.) If \( f'(c) = 0 \) and \( f(c) \) is a relative minimum, what sign must \( f''(c) \) have?

b) (2 pts.) Draw a picture of a continuous function \( f \) where \( f(c) \) is a relative maximum and \((c, f(c))\) is a point of inflection.

9. (4 pts.) Find the horizontal asymptotes of the function \( f(x) = \frac{5}{1 + 3e^{-x}} \).
Scrap: