Differentiate each of the following:

1. \( f(x) = 3^2 + x^2 + 3^x \)
2. \( f(x) = \pi^2 x^2 + 2x + \pi \)
3. \( f(x) = \frac{3}{\sqrt{x^2}} \)
4. \( f(x) = (x^2 + 5)(x^5 - 3x + 2) \)
5. \( f(x) = \frac{2x - 1}{x^2 + 1} \)
6. \( f(x) = \frac{x^3}{x^2 + 25} \)
7. \( f(x) = (3x + 1)^6 \)
8. \( f(x) = e^{2x^3 + 1} \)
9. \( f(x) = \ln(x^2 + e^x) \)
10. \( f(x) = \log(x^2 + e^x) \)
11. \( f(x) = e^{2x} \ln((x - 1)^2) \)
12. \( f(x) = 10^{2x} \log((x - 1)^2) \)
13. \( f(x) = (\sqrt{x} - 5)^{-\frac{5}{2}} \)

14. Find the critical values and the intervals on which \( f(x) - 3x^4 + 8x^3 + 5 \) is increasing or decreasing. Draw a graph. Must \( f' \) change sign at a critical value?

15. The derivative of \( f \), \( f'(x) \) is given in a factored form, and \( p(x) \) is a function that is always positive. Find all critical values of \( f \), the largest open intervals on which \( f \) is increasing, the largest open intervals on which \( f \) is decreasing, and all relative maxima and all relative minima. Using the additional information given for each function, sketch a rough graph of \( f \). \( f'(x) = p(x)(x - 1)(x - 3)(x - 5)^2 \), \( f(1) = 1, f(3) = -2, f(5) = 5 \)

16. Estimate all relative maxima, relative minima, inflection points, absolute maximum, and absolute minimum of the function plotted below. (so there’s a picture here....)

17. If the revenue function for a firm is given by \( R(x) = -x^3 + 36x \), find the value of \( x \) at which the revenue is maximized.

18. Suppose the second derivative of some function \( y = f(x) \) is given as \( f''(x) = (x - 1)(x - 2)^2(x - 3) \). Find the intervals where the graph of the function is concave up and where it is concave down. Find all inflection values.
19. Suppose the demand curve is given by \( x = 24 - 3p \).
   
   (a) Find \( E \), the elasticity of demand.
   (b) Determine whether the demand is elastic, inelastic or neither at the following price levels:
      
      i. \( p = \$3 \)
      ii. \( p = \$4 \)
      iii. \( p = \$5 \)
   (c) At what price and quantity levels is the revenue maximized?

20. Find the second derivative for the following:
   
   (a) \( f(x) = 3^2 \)
   (b) \( f(x) = \sqrt{x^2} \)
   (c) \( f(x) = (3x + 1)^6 \)

21. Find \( \lim_{x \to \infty} \frac{x^2}{x^3 + 1} \)

22. Find \( \lim_{x \to \infty} (1 + 2e^{-x}) \)

23. Find two nonnegative numbers \( x \) and \( y \) with \( 2x + y = 30 \) for which the term \( xy^2 \) is maximized.

24. A package mailed in the United States must have the length plus girth not exceed 108 inches. Find the dimensions of a rectangular package with square base of greatest volume that can be mailed. (The girth is the perimeter of a rectangular cross section.)

25. A farmer wishes to enclose a rectangular field of area 200 square feet using an existing wall as one of the sides. The cost of the fence for the other three sides is $1 per foot. Find the dimension of the rectangular field that minimizes the cost of the fence.

26. A hotel has 10 luxury units that it will rent out during the peak season at $300 per day. From experience management knows that one unit will become vacant for each $50 increase in charge per day. What rent should be charged to maximize revenue?

27. Find the most economical proportions (in the sense of minimizing the material needed) for a closed cylindrical can (ie, a cylindrical can including the top and bottom) that will hold 16 cubic inches.

28. Find the slope of the curve given by \( xy^2 + xy + y^3 = 5 \), at the point \( (2, 1) \).

29. Suppose the demand \( x \) and the price \( p \) are related by the equation \( 16x^2 + 100p^2 = 500 \). Find \( \frac{dx}{dp} \) at the point where \( p = 1 \).

30. Suppose the demand quantity and demand price are fluctuating with time, but are always related by the relationship \( 3x + xp = 32 - p^2 \) where the units of \( x \) are in millions and the units of \( p \) are in dollars. Find \( \frac{dp}{dt} \) when \( x = 2 \), \( p = 4 \) and \( \frac{dx}{dt} = .25 \).

31. A ship is observed to be 4 miles due north of port and traveling due north at five miles per hour. At the same time another ship is observed to be three miles due west of port and traveling due east on its way back to port at 4 miles per hour. What is the rate at which the distance between the ships is changing?