Math 2410, Answer to Quiz 3 (2/16/09)

There are 2 questions. All answers have to be accompanied by supporting calculation or reasoning.

(1) Draw the bifurcation diagram for the equation

\[ \frac{dy}{dt} = y^2(y + \mu) . \]

Fill in a few representative phase lines to indicate the stability of the equilibrium solutions. At what value of \( \mu \) does bifurcation occur?

**Answer:** The equilibrium solution are solution of \( y^2(y + \mu) = 0 \). In other words, \( y = 0 \) or \( y = -\mu \). In the \( (\mu, y) \) plane, they can be represented by two straight lines \( y = 0 \) and \( y = -\mu \). When \( \mu = 0 \), bifurcation occurs. The number of equilibrium solutions changes from 2 to 1 and back to 2 again. (See figure 1).

(2a) Find the general solution to the linear homogeneous equation \( \frac{dy}{dt} - y = 0 \).

**Answer:** The general solution to the homogeneous equation is \( y_h = Ce^t \) for any arbitrary constant \( C \).

(2b) Use the method of undetermined coefficient to find a particular solution to the linear non-homogeneous equation \( \frac{dy}{dt} - y = 3e^{2t} \). Then find its general solution.

**Answer:** A particular solution will be of the form \( y_p = Ae^{2t} \). Putting it into the non-homogeneous equation,

\[ 2Ae^{2t} - Ae^{2t} = 3e^{2t} \]

which gives \( A = 3 \). Thus the general solution for the non-homogeneous equation is:

\[ y = y_p + y_h = 3e^{2t} + Ce^t \]

for any constant \( C \).
Figure 1: bifurcation diagram for $\frac{dy}{dt} = y^2(y + \mu)$. 