There are 2 questions. All answers have to be accompanied by supporting calculation or reasoning.

(1) Solve the initial value problem

\[
\begin{cases}
\frac{dy}{dt} = \frac{t^2}{y(1+t^3)}, \\
y(0) = -2.
\end{cases}
\]

You may need to use the method of substitution to integrate. Express \( y \) as a function of \( t \) in your final answer.

*Answer:* Integrating \( y \, dy = \frac{t^2}{1+t^3} dt \), we have

\[
\frac{y^2}{2} = \frac{1}{3} \log |1+t^3| + C.
\]

To satisfy the initial condition, we require \( C = 2 \). Hence \( y^2 = \frac{2}{3} \log |1+t^3| + 4 \). Consequently,

\[
y = -\sqrt{\frac{2}{3} \log |1+t^3| + 4}.
\]

Note that we select the negative sign for square root, because we know that \( y(0) = -2 \).

(2) Consider the equation \( \frac{dy}{dt} = (y-1)(y-2) \).

(a) Find all its equilibrium solutions.

*Answer:* Setting \( \frac{dy}{dt} = 0 \) to find the equilibrium solutions, we have \((y-1)(y-2) = 0\). Thus \( y = 1 \) and \( y = 2 \) are the equilibrium solutions.
Figure 1: slope field for $dy/dt = (y - 1)(y - 2)$

(b) Draw the slope field and a few representative solution trajectories representing different initial conditions in the $(t, y)$ plane.

Answer: First draw the lines $y = 1$ and $y = 2$, which represent the two equilibrium solutions. Then for the solution curves that start with initial conditions $y(0) > 2$, $1 < y(0) < 2$ and $y(0) < 1$, respectively. The plot is given in figure 1.