(1) (20 pts) Solve the equation $y' = (1 + t^2)e^{-2y}$ subject to the condition $y(0) = 1$. Express $y$ explicitly as a function of $t$. 
(2) Consider the equation
\[ \frac{dy}{dt} = (y^2 - \mu)(y^2 - 4) \]

(a) (10 pts) Draw the bifurcation diagram. (Use your graphing calculator to help if necessary.)

(b) (6 pts) Put arrow directions in representative phase lines to indicate the stability of the steady state solutions.

(c) (4 pts) At what values of \( \mu \) will bifurcation occur. Explain
(3) (20 pts) Given the system

\[
\frac{dx}{dt} = y, \\
\frac{dy}{dt} = x^2 + t + 1,
\]

with initial condition \(x(0) = 1\) and \(y(0) = 0\). Write down the general formula for the Euler’s method. Use it with a step size \(\Delta t = 0.1\) to find the approximate solution for \(x\) and \(y\) at \(t = 0.2\).
(4) (20 pts) Using the method of undetermined coefficients, find the general solution to the equation

\[ \frac{dy}{dt} = y + \cos 2t. \]
(5) (20 pts) Using the method of integrating factor, find the solution to

\[
\frac{dy}{dt} + 2ty = t
\]

with initial condition \( y(0) = 1 \).