Math 211, Answer to Quiz 5 (3/19/08)

(1) (10 pts) Find the eigenvalues and the corresponding eigenvectors of the matrix A, where

$$A = \begin{pmatrix} 1 & 3 \\ 5 & 3 \end{pmatrix}.$$

Answer: We have

$$A - \lambda I = \begin{pmatrix} 1-\lambda & 3 \\ 5 & 3-\lambda \end{pmatrix}.$$

To find the eigenvalues of A, we set $det(A - \lambda I) = 0$. This simplifies to $\lambda^2 - 4\lambda - 12 = 0$. Hence the eigenvalues are $\lambda_1 = 6$ and $\lambda_2 = -2$.

When $\lambda_1 = 6$, we solve

$$(A - \lambda_1 I)\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -5 & 3 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

and get $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix}$. This is an eigenvector for $\lambda_1 = 6$. Similarly we obtain eigenvector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ for $\lambda_2 = -2$.