Math 211, Answer to Quiz 2 (2/6/08)

(1) Consider the equation

$$\frac{dy}{dt} = y(y-1)(y+2)$$

(a) (4 pts) What are the equilibrium (steady state) solutions for the above equation.

Answer: $y = -2$, $y = 0$ and $y = 1$ are the equilibrium solutions.

(b) (6 pts) Sketch its slope fields.

Answer: You should draw a picture of the slope field. In the y versus t plane,

(a) For $y < -2$, the slope field goes down (negative slope).
(b) For $-2 < y < 0$, the slope field goes up.
(c) For $0 < y < 1$, the slope field goes down.
(d) For $1 < y$, the slope field goes up.

(2) (10 pts) Consider the equation

$$\frac{dy}{dt} = t - y^2 \text{ with } y(0) = 1.$$

Use the Euler's method with a step size $\Delta t = 1/2$ to find the approximate value of $y(1)$.

Answer: We have $\Delta t = 1/2$, $Y_0 = 1$, $t_k = k\Delta t = k/2$. Use the Euler's method formula:

$$Y_{k+1} = Y_k + \Delta t\ (t_k - Y_k^2)\ , \quad k = 0, 1, \ldots .$$

Thus

$$Y_1 = 1 + \frac{1}{2}(0 - 1^2) = 0.5\ ,$$

and

$$Y_2 = .5 + \frac{1}{2}(.5 - .5^2) = 0.625\ .$$

Since $t_2 = 1$, thus Y_2 is an approximation to $y(1)$. Hence $y(1) \approx 0.625$.