

**Math 211, Answer to Quiz 1 (1/30/08)**

(1) (10 pts) Solve the initial value problem

$$\begin{cases} \frac{dy}{dt} = \frac{yt}{1+t^2}, \\ y(0) = 1. \end{cases}$$

Express  $y$  explicitly as a function of  $t$  in your final answer.

*Answer.* We have

$$\int \frac{dy}{y} = \int \frac{t}{1+t^2} dt$$

Using the substitution  $u = 1 + t^2$  on the right hand side, one can check that

$$\ln|y| = \frac{1}{2}\ln(1+t^2) + C = \ln\sqrt{1+t^2} + C$$

for some arbitrary constant  $C$ . In other words,

$$|y| = e^C \sqrt{1+t^2}.$$

Or,  $y = k\sqrt{1+t^2}$  for some constant  $k$ .

Putting in the initial condition  $y(0) = 1$ , we have  $k = 1$ . Thus

$$y = \sqrt{1+t^2}$$

is the required solution.