Name: _____

Math 211 Exam 1 Feb, 27, 2007

(1) Consider the equation

$$\frac{dy}{dt} = -y(y-1)(y+2)$$

(a) (3 pts) What are the equilibrium (steady state) solutions for the above equation.

(b) (6 pts) Sketch the slope fields and, to its right, the phase lines for the equation.

(c) (3 pts) Indicate which equilibrium points are stable or unstable. Explain.

(d) (3 pts) What happens as $t \to \infty$ if the initial condition is $y(0) = 1/2$?

(2a) (10 pts) A 10-gallon bucket is full of pure water. Suppose we begin dumping salt into the bucket at a rate of 2 pounds per minute. Also a faucet at the bottom of the bucket is opened to let out 2 gallons of salted water per minute. At the same time, we add pure water to keep the bucket full. If the salt water is always well mixed, formulate a governing differential equation on $S(t)$, which is the amount of salt inside the bucket at time t. What is the initial condition on S at $t = 0$? Explanation is essential.

(2b) (10 pts) Solve the equation $dy/dt = 5 - 2y$ subject to $y(0) = 1$ using the method of separation of variable.

(3) Consider the equation

$$\frac{dy}{dt} = y^3 - 9y - \mu$$

(a) (9 pts) Draw the bifurcation diagram. (Use your graphing calculator to help.)

(b) (4 pts) Put arrow directions in representative phase lines to indicate the stability of the steady state solutions.

(c) (2 pts) At what value of μ will bifurcation occur. Explain

(4) (15 pts) Given the system

$$\begin{cases} \dfrac{dx}{dt} &= -2y \ , \\ \dfrac{dy}{dt} &= x^2 + y \ . \end{cases}$$

with initial condition $x(0) = 1$ and $y(0) = -2$. Write down the general formula for the Euler's method. Use it with a step size $\Delta t = 0.5$ to approximate $y(1)$. What is this approximate value?

(5a) (10 pts) Using the method of undetermined coefficients, find the solution to the equation

$$\frac{dy}{dt} = 3y + e^{2t}$$

with initial condition $y(0) = 0$.

(5b) (10 pts) Solve the same problem in (5a) using the method of integrating factor. Check that the solutions are the same.

(6a) (8 pts) Convert the second order equation

$$\frac{d^2x}{dt^2} + x(\frac{dx}{dt})^2 - 1 + x^2 = 0$$

into a first-order system in term of x and v, where $v = dx/dt$.

(6b) (7 pts) Find all the equilibrium points of the resulting first-order system.